Sinusoidal Steady State

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Purpose:

Examine the sinusoidal steady-state response of a resonant circuit in the phasor domain.

Equipment Required:

- 1 - Agilent 34401A Digital Multimeter
- 1 - Agilent 33120A Waveform Generator
- 1 - Agilent 54622A Deep Memory Oscilloscope or Agilent 54600B Oscilloscope
- 1 - Protoboard
- 1 - 1-kΩ Resistor
- 1 - 0.1-µF Capacitor
- 1 - 10-mH Inductor

Prelab:

Review Section 13.6 in the text*.

a. Find the s-domain driving-point impedance $Z_{EQ}(s)$ and the voltage transfer function $T_V(s)$ for the circuit of Fig. 1. Find the undamped natural frequency ω_O and the damping ratio ζ.

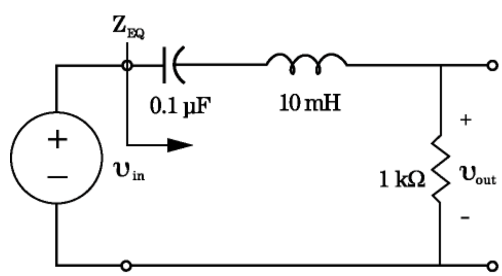

Figure 1

b. In your journal, draw the circuit of Fig. 1 labeling the elements with their impedance (see Eq 13-19 in the text for the impedance equations). Find the phasor domain driving-point impedance $Z_{EQ}(j\omega)$, the frequency-response gain function $|T_V(j\omega)|$ and the phase response $\theta(\omega) = \angle T_V(\omega)$ for the circuit of Fig. 1. Compute the resonant frequency, f_O.

Procedure:

1. Prepare a data table and build the circuit

   a. Prepare a data table with four columns, labeled frequency, v_{in}, v_{out}, and phase angle q. Starting at a frequency of 50 Hz, fill in the frequency column in standard logarithmic frequency steps (1, 2, 5, 10 etc.) just short of the computed resonant frequency, f_O. Mark the next line in the data table with the notation “resonance,” then continue in standard logarithmic frequency steps for two more decades.

b. Build the circuit of Fig. 1 on a protoboard. Set the waveform generator to produce a 2 V_{pp} sine wave (V_A = 1 V), and no DC offset (V_{avg} = 0).

2. The Lissajous pattern

A *Lissajous pattern*, Fig. 2, can be used to measure the phase angle θ between two sinusoids of the same frequency. The following procedure provides details on how to generate the Lissajous pattern and measure the phase angle.

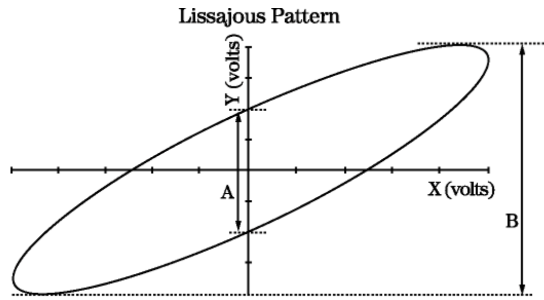

Figure 2

a. Connect v_{in} to the X channel of the scope, and v_{out} to the Y channel.

b. Set the oscilloscope to Main/Delayed and choose the X-Y mode.

c. Set the VOLTS/DIV on the X and Y channels so that the Lissajous pattern is as large as possible, but is completely visible on the CRT. At frequencies other than resonance, the Lissajous pattern will be similar that of Fig. 2.

d. Verify that the Lissajous pattern is still centered on the oscilloscope CRT. If not, use the position controls to center the pattern on the CRT.

e. Measure the two distances A and B in Fig. 2 using the cursor controls. The dimension A is the distance between the two intersections of the Lissajous pattern and the Y axis. B is the vertical distance between the top and the bottom of the pattern.

f. Compute the phase angle θ from Eq. 1.

$$\theta = \sin^{-1}\left(\frac{A}{B}\right) \qquad (1)$$

The sign of the phase angle θ cannot be determined directly from the Lissajous pattern. To find the sign of θ, set the scope in normal time-base mode (not X-Y), and identify whether the output waveform leads or lags the input. Locate the positive-going zero crossing for the input signal and the nearest positive-going zero crossing for the output signal. If the output zero crossing occurs after the input, as shown in Fig. 3, the output is said to “lag” the input, and θ is negative. If the output waveform crosses zero before the input waveform does, the output is said to “lead” the input, and θ is positive.

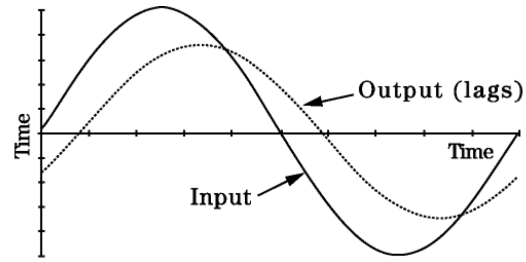

Figure 3

At resonance, the reactance X is zero and the impedance is purely resistive. Since the imaginary portion of the impedance is zero, $Im\{T_V(j\omega)\} = 0$ and for this circuit the phase angle $\theta = 0$. All frequencies above resonance have the same sign, and frequencies below the resonant point have the opposite sign. Verify that this is true over a frequency range two orders of magnitude on both sides of the resonant frequency.

3. Collect the data

   a. Locate the actual resonant frequency f_O by tuning the frequency until the Lissajous pattern is a single diagonal line. Record the frequency indicated by the waveform generator. Record the measured phase angle at resonance.

   b. For each of the remaining frequencies in the data table measure the phase angle between v_{in} and v_{out}.

   c. Configure the oscilloscope to display v_{in} and v_{out} (normal time-base mode, not X-Y). Set the frequency to match each phase-angle data point collected in procedure 3b and at each of these points measure and record the peak-to-peak values of v_{in} and v_{out}.

Conclusion

a. In your journal, provide two graphical representations in the complex plane of the impedance Z_{EQ} for the circuit in Fig. 1. For the first drawing use a frequency of $f_O/2$. Draw in Z_L and Z_c along the imaginary axis, and Z_R along the real axis. Draw in Z_{EQ}, which is the vector sum of Z_L, Z_C and Z_R. Label the magnitude and phase angle for Z_{EQ}. Repeat this procedure for a second frequency, $2f_O$. Review Appendix C, including Fig. C-3, in the text for details on arithmetic addition of complex numbers.

b. Use a spread sheet program to calculate the theoretical gain and phase angle data for the circuit of Fig. 1. Remember that vector addition must be used to compute the theoretical gain and phase responses. Enter the v_{in} (pp), v_{out} (pp) and phase data from your data table into the spreadsheet. Compute the gain $|T_V(j\omega)| = v_{out}/v_{in}$ from your empirical peak-to-peak data. Generate and print out Bode plots for the circuit of Fig. 1. (Review Sect 12-5 for details on Bode plots). Use a solid line to plot the empirical data, and a dotted line for the theoretical data. Tape both the gain response and phase response plots into your lab journal.

[*] Roland E. Thomas and Albert J. Rosa, The Analysis and Design of Linear Circuits, Prentice Hall, (New Jersey, 1994)